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(a) Find the intervals of increase or decrease.
(b) Find the local maximum and minimum values.
(c) Find the intervals of concavity and the inflection points.
(d) Use the information from parts (a), (b), and (c) to sketch the graph. Check your work with a graphing device if you have one.

27. $f(x) = 2x^3 - 3x^2 - 12x$

28. $f(x) = 2 + 3x - x^3$

29. $f(x) = x^4 - 6x^2$

30. $g(x) = 200 + 8x^3 + x^4$

31. $h(x) = 3x^5 - 5x^3 + 3$

32. $h(x) = (x^2 - 1)^3$

33. $P(x) = x\sqrt{x^2 + 1}$

34. $P(x) = x\sqrt{x + 1}$

35. $Q(x) = x^{1/3}(x + 3)^{2/3}$

36. $Q(x) = x - 3x^{1/3}$

37. $f(\theta) = \sin^2\theta, \quad 0 \leq \theta \leq 2\pi$

38. $f(t) = t + \cos t, \quad -2\pi \leq t \leq 2\pi$

Assignment #4

1. Find the absolute minimum and maximum values of the function, if they exist, over the given interval.

   a) $f(x) = x^2-6x-3$; $[-1, 5]$

   b) $f(x) = x^3-3x+6$; $[-1, 3]$

   c) $f(x) = 2x^3-3x^2-36x+62$; $[-3, 4]$

   d) $f(x) = x + \frac{1}{x}$; $[1, 20]$

   e) $f(x) = \frac{x^2}{x^2 + 1}$; $[-2, 2]$

   f) $f(x) = \frac{x}{(x + 9)^2}$; $[-1, 8]$

   g) $f(x) = -3$; $[-2, 2]$

2. An employee's monthly production M, in number of units produced, is found to be a function of the number of year of service, t. For a certain product, a productivity function is being given by:

   $$M(t) = -2t^2+100t+180,\ 0 \le t \le 40$$

   Find the maximum productivity and the year in which it is achieved.

3. A firm determines that its total profit in dollars from the production and sale of x units of a product is given by:

   $$P(x) = \frac{1500}{x^2 - 6x + 10} \qquad x \ge 0$$

   Find the number of units x for which the total profit is a maximum.

4. The temperature, T, of person during an illness is given by:

   $T(t) = -0.1t^2 + 1.2t + 98.6;\ 0 \le t \le 12,$

   where T = temperature (°F) at time t, in days.

   Find the maximum value of the temperature and when it occurs.

5. Technicians working for the Ministry of Natural Resources have found that the amount of a pollutant in a certain river can be represented by where t is the time (in years) since a clean up campaign started. At what time was the pollution at its lowest level?

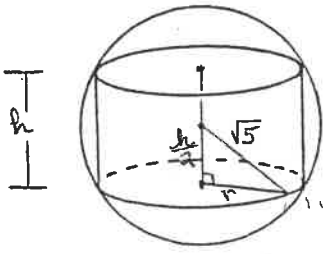
   $$P(5) = 2t + \frac{1}{162t + 1} \qquad 0 \le t \le 1$$

1. A rectangle with sides parallel to the coordinate axes is inscribed in the region enclosed by

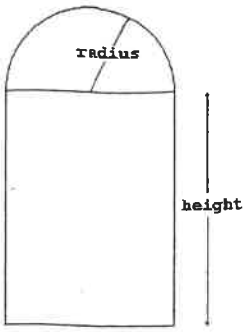
$y = -4x^2 + 4$ and the x-axis.

Find the coordinates of the point on the function $y = -4x^2 + 4$ so the area of the rectangle is a maximum.

2. Find the height and radius of the largest right circular cylinder that can be placed into a sphere of radius $\sqrt{5}$.



3. A Norman window consists of a rectangle surmounted by a semicircle. If the perimeter (both the rectangle and the semicircle together) of a Norman window is to be 32 ft, determine what should be the radius of the semicircle and the height of the rectangle such that the window will admit the most light (have the most area).



4. A trough is to be made that is 20 ft in length with ends that form isosceles trapezoids. The isosceles trapezoid end sections each have a base, one foot in length, with legs sloping outward that are one foot in length. θ is the angle, that marks how much the sides of the trough are sloping outward from a vertical alignment. What value of θ will give the trough its maximum volume?

Assignment Sheet #5

Area and Volume Problems

1. A farmer has 54 m of fencing with which to build two animal pens with a common side. One pen is rectangular; the other is square. If the area of the pens is to be maximized, what are their dimensions?
2. A right circular cone is inscribed in a sphere of radius 15 cm. Find the dimensions of the cone that has the maximum volume. The volume of a cone is $V = \frac{\pi r^2 h}{3}$.
3. A printed page of total area 320 cm^2 has top and side margins of 2 cm and a bottom margin of 3 cm. Find the dimensions of the page that make the area of print a maximum.
4. Find the length of the sides of the isosceles triangle of greatest area that has a perimeter of 18 cm.

Time and Distance Problems

1. Find the point on the parabola $x = y^2 - 8y + 18$ closest to $(-2,4)$.
2. A pair of scuba divers wish to dive on a wreck that lies 0.4 km off the shore from a point 0.8 km from their present position. If they can walk carrying their gear at 5 km/h and swim at 3 km/h. What course should they follow to reach the wreck in the minimized time?
3. A light airplane flying east at 300 km/h passes over Buttonville airport 10 min before a second airplane flying southeast 420 km/h passes over the same point. Assuming that both the airplanes are at the same altitude, what is the minimum distance between the planes?

Profit Problems

Notes: total revenue + number of units • price per unit
total profit + Revenue - cost of production

1. A stereo manufacturer determines that in order to sell x units pf a new stereo, the price per unit must be $p = 1000 - x$. The manufacturer also determines that the total cost of producing x units is given by $C(x) = 3000 + 20x$.
   a) find the total revenue $R(x)$
   b) find the total profit $P(x)$
   c) how many units must the company produce and sell in order to maximize profit?
   d) what is the maximum profit?
   e) what price per unit must be charged in order to make this maximum profit?
2. Raggs Ltd. A clothing firm, determines that in order to sell x suits, the price per suit must be $p= 150 - 0.5x$. It also determines that the total cost of producing x suits is given by $C(x) = 4000 + 0.25x^2$.
   a) find the total revenue $R(x)$
   b) find the total profit $P(x)$
   c) how many suits must the company produce and sell in order to maximize profit?
   d) what is the maximum profit?
   e) what price per suit must be charged in order to make this maximum profit?

A.P. Calculus Maxima Minima Problems

1. Determine the maximum product of two positive numbers whose sum is 8.

2. Determine the maximum area of a rectangle that can be enclosed with 200 meters of fence.

3. Determine the minimum area of a poster that will contain 50 square inches of printed material and have 4 inch margins on the top and bottom and 2 inch margins on the left and right.

4. Determine the dimensions of a box of maximum volume that can be made from a piece of material 8" x 10". The box is to be made by cutting square pieces from the corners and folding up the sides. The box will not have a top.

5. Determine the maximum area of a rectangle with one side on the x-axis and the opposite corners touching the parabola $y = -x^2 + 9$.

6. A rectangle is to be inscribed in a right triangle having sides 6 inches, 8 inches, and 10 inches. Determine the dimensions of the rectangle with greatest area.

7. Given a length of string L = 50 inches, construct a circle and a square such that the sum of the areas is a maximum.

8. This is the infamous *Question 6* on the 1982 A.P. Exam which was the focus of the movie, *Stand and Deliver*.

   A tank with a rectangular base and rectangular sides is to be open at the top. It is to be constructed so that its width is 4 meters and its volume is 36 cubic meters. If building the tank costs $10 per square meter for the base and $5 per square meter for the sides, what is the cost of the least expensive tank?